Generating Multistep Methods for Special Ordinary Differential Equations of Higher-Order

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ABSTRACT

It is possible to integrate a differential equation of higher-order of the form $y^{(n)} = f(x, y, y', y'', ..., y^{(n-1)})$ by reducing it to the system $y' = y_1$, $y_1' = y_2$, ..., $y_{n-2}' = y_{n-1}$, $y_{n-1}' = f(x, y, y_1, y_2, ..., y_{n-1})$, and applying one consistent and stable method for systems of equations of higher order. This procedure is a perfectly legitimate one. No accuracy is lost, nor is there any unnecessary outlay of computional effort.

The situation is slightly different if the equation to be integrated is of the form $y^{(n)} = f(x, y)$, $n > 2$, where no derivates appear in the right-hand side member of the differential equation. Equations of this type are called *special* differential equations. If one is not particularly interested in the values of the intermediate derivates, it seems unnatural to introduce them artificially in order to produce systems of first order equations.

Therefore, it is interesting to have consistent and stable methods to solve directly these equations. In this paper, it is proposed a general framework for multistep methods for special ordinary differential equations of higher-order of the form $\sum_{i=0}^{k} \alpha_i y_{n+i} = h^n \sum_{i=0}^{k} \beta_i f_{n+i}$. This framework is such that a method for a equation of order n has as its α coefficients the coefficients of the $(x-1)^n$ Newton's Binomial ($\alpha_i = (-1)^k C_{n,k}$), and $\sum_{i=0}^{k} \beta_i = 1$. This family of methods can be proved consistent and stable. Moreover, the βs can be adjusted so that the consistence order can be increased and the error constant, decreased.

References

[1] Atikinson, Kendall E., An Introduction to Numerical Analysis, 2^{nd} Edition, John Wiley & Sons (1988)

[2] Boyce, Willian E., Prima, Richard C. Di, Elementary Differential Equations and Boundary Value Problems, 5^{th} Edition, John Wiley & Sons (1992)

[3] Costa, Therezinha Souza, Lecture Notes of Numerical Methods for Differential Equations, In Portuguese

[4] Henrici, Peter, Discrete Variable Methods in Ordinary Differential Equations, John Wiley & Sons (December 1962)

[5] Imperial, Juliana Carpes, An Overview of the Consistence Analysis, Monography, Departamento de Informática, PUC-Rio, In Portuguese (2001)

[6] Lambert, J. D., Computional Methods in Ordinary Differential Equations, John Wiley & Sons (1972)

[7] Ruggiero, Márcia A. Gomes, Lopes, Vera Lúcia da Rocha, Numerical Calculus, 2^{nd} Edition, Makron Books, In Portuguese (1996)

[8] Simmons, George F., Calculus with Analytic Geometry, Volume 1, McGraw-Hill 1987