

Analysing Methods to Solve Ordinary Differential Equations

Juliana Carpes Imperial
Departamento de Informática
Pontifícia Universidade Católica do Rio de Janeiro (PUC-Rio)
Rua Marquês de São Vicente, 225, Gávea, 22.453-900, Rio de Janeiro, RJ, Brazil
juliana@inf.puc-rio.br

ABSTRACT

This work presents a program which makes an analysis of the consistence and the stability of multistep methods for ordinary differential equations. The methods can be simple or system of methods, which can be just composite or cyclic.

The consistence is very important for a correct approximation of the solution of a differential equation. An inconsistent method may seem to converge to a supposed solution, which is actually an incorrect approximation of it. For systems of methods, the analysis of the consistence is only made if it is cyclic and if it is for ordinary differential equations of the first order. However, for simple methods, the analysis of the consistence is also made if the method is for special ordinary differential equations of higher-order. Moreover, for composite and cyclic methods for ordinary differential equations of the first order, the real order of consistence, in some cases, can be shown to be 1 plus the minimum consistence order of the methods which are part of the system. This was implemented only for cyclic methods.

To analyse the stability, the exponential test is done. That is, the method is tested with the differential equation of the form $y^{(n)}(x) = y(x)$, $y(0) = 1$, $y(0)' = 1$, ..., $y(0)^{(n-1)} = 1$, where n is the order of the differential equation. This test is only done if the method is consistent and if the program calculates it. Since the method is known to be consistent, the method will not converge to a wrong solution if it is stable. Therefore, if the exponential test converge, the method is stable.

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